Santa Claus: Formal Analysis of a Process-Oriented Solution

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With the commercial development of multicore processors, the challenges of writing multi-threaded programs to take advantage of these new hardware architectures are becoming more and more pertinent. Concurrent programming is necessary to achieve the performance that the hardware offers. Traditional approaches present concurrency as an *advanced* topic: they have proven difficult to use, reason about with confidence, and scale up to high levels of concurrency. This paper reviews *process oriented design*, based on Hoare's algebra of Communicating Sequential Processes (CSP), and proposes that this approach to concurrency leads to solutions that are manageable by novice programmers – that is, they are easy to design and maintain, that they are scalable for complexity, *obviously correct*, and relatively easy to verify using formal reasoning and/or model checkers. These solutions can be developed in conventional programming languages (through CSP libraries) or specialised ones (such as occam-π) in a manner that directly reflects their formal expression. Systems can be developed without needing specialist knowledge of the CSP formalism, since the supporting mathematics is burnt into the tools and languages supporting it. We illustrate these concepts with the *Santa Claus Problem*, which has been used as a challenge for concurrency mechanisms since 1994. We consider this problem as an example control system, producing external signals reporting changes of internal state (that model the external world). We claim our occam-π solution is *correct-by-design*, but follow this up with formal verification (using the FDR model checker for CSP) that the system is free from deadlock and livelock, that the produced control signals obey crucial ordering constraints, and that the system has key liveness properties.

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1. INTRODUCTION

1.1 Concurrency is our Friend

In the teaching of computer science, concurrency is commonly treated as an 'advanced' topic – only to be approached, if at all, once students have learned and become comfortable with sequential programming. In the practice of computer science, concurrency is commonly engaged with only as a last resort [Muller and Walrath 2000] – to deal with performance issues (such as response latencies in an embedded control system or the efficient use of a parallel supercomputer).

These mindsets have been formed by painful experience over five decades. Concurrency just causes too many surprises. In contrast with sequential logic, system state we thought we had under one thread of control can get hit by another – sometimes – and the result is chaos. So, we add protection in the form of locks and, should we get this wrong, the system deadlocks ... or livelocks ... or parts of it get starved of attention. Obviously, concurrent logic will be much harder than sequential stuff?

No. Concurrency is fundamental to the workings of the universe. It exists at all levels of granularity (e.g. nanoscale, human, astronomic). Complex, interesting and useful behaviour emerges from the concurrent actions of zillions of processes, each managing its own – *and only its own* – state, and synchronising and communicating to enable and/or constrain each others' individual behaviours.

To provide useful service to its environment, a computer system needs to reflect that environment. That environment, being part of the natural world, is concurrent. The computer system, *for simplicity* therefore, needs to be concurrent.

A default restriction of programmed logic to sequential design and implementation is unnatural. A penalty is the increasing difficulty of managing complex behaviour. In an aerodynamically unstable airplane, we need to control *both* wings *and* the tail fin all at the same time! Programming the necessary logic in one thread of control is asking for trouble. Programming it as a network of *Communicating Sequential Processes* directly matches the problem structure and will be (much) easier. Richer complexity can be built through layers of network – just as in real life. Welcome to CSP.

The above thesis has lain around for nearly thirty years, which is about the time major ideas take to mature and be applied in computer science. Performance issues have (wrongly) been the driving force for concurrency to date. The commercial arrival of multicore processors – themselves forced on us by the laws of physics – brings that driving force into play immediately. On a quad-core processor, sequential code cannot use more than 25% of the processing power. If we need more, there has to be concurrently executable code - and lots of it. To be in with the chance of keeping the cores busy with useful work, there needs to be (an order of magnitude) more concurrency in the software than is available in the hardware – the principle of *parallel slackness* first discussed by Valiant [Valiant 1990].

There is another problem here. It is a suspicion that most existing multithreaded applications have concurrency errors. Many of these errors remain hidden because of favourable scheduling sequences on unicore processors. On multicores, with real parallel execution, any such errors have a greater risk of causing trouble. People are scared – but there is no avoiding it.

This paper reviews and demonstrates the use of the concurrency model outlined above and inspired by Hoare's CSP process algebra. The good news is that there will be no maths here. The even better news is that there are elegant and powerful mathematical properties (e.g. *compositionality*) underlying the model and built into the languages, libraries and tools supporting it – we get the benefits simply by using them. As well as simplification in application logic, safe and efficient exploitation of multicore processors follows automatically At least, this is true for the occam-π multiprocessing language [Barnes and Welch 2004; Welch and Barnes 2005a; 2005b; 2008; Ritson et al. 2009; Sampson et al. 2010; Barnes et al. 2010], used in this paper, and the JCSP [Welch 2000; Welch et al. 2007; Welch and Austin 2010] and C++CSP [Brown and Welch 2003; Brown 2007] libraries for Java and C++.

1.2 The Santa Claus Problem

For illustration, we consider a problem first suggested by John A. Trono [Trono 1994]. This problem is known as *The Santa Claus Problem.* Trono's description, with a few added words, is as follows:

> Santa repeatedly sleeps until wakened by either all of his nine reindeer (back from their holidays) or by a group of three of his ten elves (who have left their workbenches). If awakened by the reindeer, he harnesses each of them to his sleigh, delivers toys with them and finally unharnesses them (allowing them to go back on holiday and him to go back to sleep). If awakened by a group of elves, he shows each of the group into his study, consults with them on toy R&D and finally shows each of them out (allowing them to go back to work and him to sleep). Santa should give priority to the reindeer in the case that there is both a group of elves and a group of reindeer waiting. Initially, the reindeer are all on holiday, the elves are at their workbenches and Santa is asleep.

Although this problem seems simple at first, it presents challenges in concurrent control that are typical of a wide range of computer application. Over the years, solutions have been published demonstrating many approaches to concurrency (Section 7.1). The first of these solutions (using semaphores [Trono 1994]) was shown to be incorrect (by Ben-Ari et al. [Ben-Ari 1998]) and replaced by one using monitors. The fact that the first published solution was wrong is evidence that the problem is not, in fact, that simple.

We can imagine the changes of state in the Santa Claus system reflecting the state of some real machine – for example, an airplane or nuclear power plant. With the addition of output signals upon state change, the system might even be controlling that machine. For many applications, that control will be safety critical and verification of its safety (*do no bad things*) and effective functioning (*do good things*) will be essential.

We extend the specification of the problem to include external reports from the system, documenting what Santa, the elves, and the reindeer are doing as the system evolves. We suppose that any machine it is controlling will break if those reports occur in certain wrong orders (see Sections 5.5 and 6). We verify that none of these can happen in our solution.

Concurrency tripwires such as race hazard, deadlock, livelock and process starvation must also, of course, be avoided. Particular safety constraints will demand that certain state changes (and their corresponding signals) must happen in certain orders – that is, that Lamport *happens-before* relations [Lamport 1978] must be enforced and verified. Liveness demands will specify that certain state changes must happen following certain events. Building solutions to such problems that are not only correct, but are seen to be correct, is a challenge for concurrent logic – though not, we claim, as big as the challenge to solve it with purely serial logic.

1.3 Paper Roadmap

This paper presents a solution that is *process-oriented* (Section 2). Concurrent design is expressed with process network diagrams (Figures 1-6) and directly refined into executable occam-π code (Sections 3 and 4). occam-π is an extended version of the classical occam [SGS-THOMSON Microelectronics Limited 1995] programming language, incorporating dynamic network construction mechanisms (channel and process mobility) from Milner's π-calculus [Milner 1999]. Formal verification of a range of correctness properties is achieved through direct mapping of the occam-π source code to CSP and model checking using FDR [Formal Systems (Europe) Ltd. 1998] (Sections 5 and 6). Switching between these representations (network diagrams, occam-π code and CSP) is straightforward, allowing maintenance to be led from any form – an example is given in Section 6.

The concurrency in our solution directly reflects the concurrency in the Santa Claus story. This *simplifies* its design and implementation, rather than making it hard. Indeed, aspects of this problem and solution have been set in formal examination for second year computer science undergraduates (at the University of Kent) and all passed. We claim that this design and development leads to solutions that are *'obviously correct'*, but that backing this up with formal verification is not hard and, probably, a good idea.

2. REVIEW OF PROCESS ORIENTED DESIGN

Process oriented design is an example of component-connector engineering. The components are active processes and the connectors are events (their *alphabets*) through which they synchronise and communicate. Key concepts are processes, channels, barriers, networks, network hierarchies, choice, protocols and synchronisation patterns. To be practical, a process oriented programming language, or a library providing the necessary support for other languages, is essential – otherwise, the gulf between the theory underpinning the design and its realisation in code presents uncomfortable obstacles. Such tools must be easy to learn and use and have reasonably efficient implementation. Fortunately, all these exist – we just have to rise to the challenge of trying them.

2.1 Processes

A *process* is a self-contained self-executing unit that encapsulates private data and algorithms. This contrasts with object oriented programming where object methods are executed by an external caller's thread of control. An object is passive (it does nothing unless a method is invoked), whereas a process is active and can take the initiative. A process has sole control over its internal resources and no control (not even visibility) of the resources of another process. Interaction with other processes happens indirectly through synchronising primitives, such as channel communication and barrier synchronisation. Crucially, a process can *refuse* some, or all, of its external events – thereby blocking demands from other processes until it is in a good state to *accept* them. An object cannot refuse a method invocation, no matter its internal state.

2.2 Synchronising Channels

The simplest form of process interaction is point-to-point synchronous unidirectional message passing along a zero-buffered *channel*. A channel has a sending end and a receiving end, though it is possible to share these between multiple senders or receivers. Zero-buffering means that a sender process must block if no receiver is ready (and vice-versa). Various kinds of channel buffering (e.g. blocking or overwriting FIFOs) can be obtained through splicing in appropriate buffer processes.

These communications differ from those in common message passing libraries for parallel computing. For example, in MPI [Dongarra 1994] **any** process knowing the process identifier of a receiving process (within one of its own communicator groups) can send it a message. In CSP, there are named process types but individual processes have no names. Individual processes are bound to a particular set of events (channels, barriers, ...) that *do* have names. Different instances of the same process type can, of course, be bound to different sets of events. A process sends to a named channel and whatever process has the other end receives. A process engaging on a named barrier blocks until whatever other processes registered for the barrier also engage. Network connectivity is explicit and dynamic and constrained to what the system needs. The difference is subtle but helps engineer high cohesion within and zero coupling between processes.

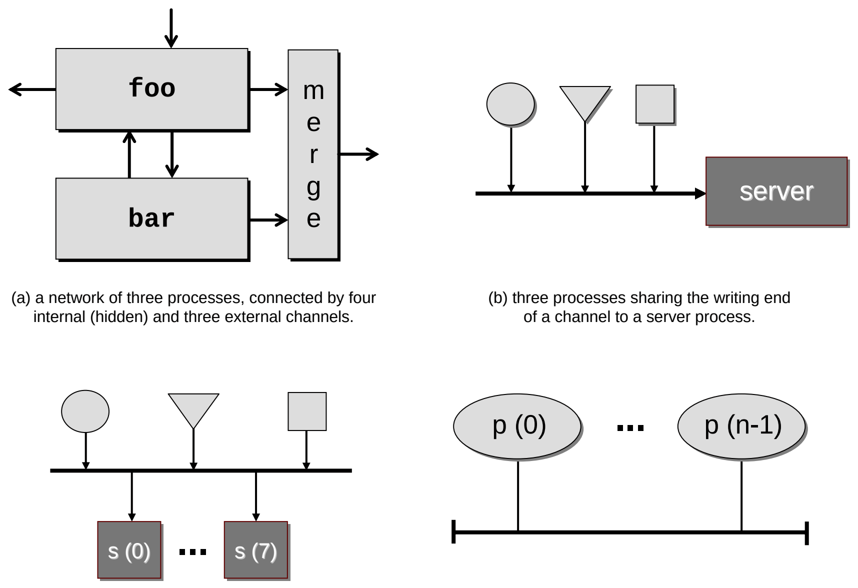
Processes cannot observe or modify each others' state, so need no locking mechanisms to maintain data integrity. To observe or modify such state, a process must communicate a request to the owning process via appropriate channels. That request may be ignored by the target process (blocking the requester) until such time as it chooses (e.g. when the request can be correctly processed). This means that reasoning about process behaviour can always be conducted *locally* – a process is in complete charge of its state.

The size of the state space of a process network is bound by the product of sizes of the state spaces of its component processes (less those that cannot be reached through the constraints of process synchronisation). Thus, the state space of a process network can grow large whilst the logic of its components remains simple. It is this gearing – a *compositional* semantics – that delivers the power of process oriented design.

In contrast, threads concurrently managing shared state through locking mechanisms (mutexes, semaphores or monitors) have to be secure in the face of all possible interleavings through the shared objects. Reasoning is *non-local*: the logic of an individual thread cannot be devised, or understood, on its own. This is hard.

2.3 Synchronising Barriers

Channels require two processes (the sender and receiver) to synchronise. A barrier is an event on which *many* processes can be enrolled and on which *all* must syn-



(a) a network of three processes, connected by four internal (hidden) and three external channels.

(b) three processes sharing the writing end of a channel to a server process.

(c) three processes sharing the writing end of a channel to a bank of servers sharing the reading end.

(d) n processes enrolled on a shared barrier (any process synchronising must wait for all to synchronise).

Fig. 1. Process oriented design: components and connectors.

chronise together. If one process offers to synchronise on a barrier, all must offer to synchronise for the event to happen – everyone must wait for everyone. A process may have any number of barriers in its alphabet.

2.4 Networks

A network is simply a parallel composition of processes (which may themselves have internal networks), connected through a set of synchronising events (channels, barriers etc.). A network usually hides the events connecting internal components, leaving free those to be used for external connections. A network is, therefore, also a process. Network topologies can be constructed dynamically and may evolve (both in shape and cardinality) in response to their environment.

2.5 Design by Pictures and Composition

Processes do not know – or need to know – with whom they are synchronising. Each process can be viewed as a *black box*, whose ties to its environment is a set of events (channels-ends, barriers, ...) – its *alphabet* in CSP language. The behaviour of a process is described by the message structures allowed on its channels, the patterns of synchronisation with which it is prepared to engage on its channels and barriers, and any computational functions it performs. Networks of processes are simply built by 'wiring' them together using internal (hidden) channels and barriers. A network is itself a process – so hierarchical structures naturally emerge.

This method of construction has an obvious visual representation, lending itself to design through (structured) pictures – see Figure 1. This should have resonance with hardware engineers, whose systems are physically concurrent. The discipline leads to a strong notion of *components* (the processes) and *connectors* (the synchronisation events), supporting concurrency, hierarchical design and code reuse. The processes run themselves and do not share memory. Innermost processes are sequential, require no locks, and engage in channels (i.e., external I/O operations) and/or barriers. They are simple and familiar (except, possibly, for the barriers) and all our skills for sequential programming remain valid.

We make use of such diagrams all the time when designing occam-π programs. The design pictures for the Santa Claus system, for which we show an occam-π implementation and verify some correctness properties, appear later in this paper.

2.6 Formal Verification

Being able to reason formally about a program is valuable – crucially so if the application is safety or finance critical. Special difficulties arise with concurrently executing processes since the state space potentially explodes. If the concurrency formalism in which reasoning is conducted differs from the implementation primitives used, the reasoning is unsafe. If translation between the implementation and formal modelling languages is hard, maintaining coherence between the two will be a continuous overhead as the system evolves.

This gap between implementation and verification is reduced by using languages (or libraries) designed around formal methods for which verification tools exist. Almost all concurrency mechanisms within occam-π have a direct representation in CSP. FDR [Formal Systems (Europe) Ltd. 1998] is a model checker for CSP, allowing formal verification of freedom from deadlock and livelock, process refinement and equivalence – at least, for systems of finite (and sufficiently small) size. FDR has a long and successful history of use in the analysis of complex safety-critical systems [Schneider and Delicata 2004; Barrett 1995; Hall and Chapman 2002; Buth et al. 1997; Buth et al. 1999; Lowe 1996; McEwan and Schneider 2007].

Translation between occam and CSP is defined, [Goldsmith et al. 1993; 1994], and can be automated. At present, we do this by hand and this paper gives an example. In general, state space introduced by real programs (for example, a single 32-bit integer variable has potentially 4 giga-values) must be reduced to small finite numbers – if the model checks are ever to terminate in acceptable times. Automating this raises several challenges that are not the subject of this paper. The Santa Claus system does not raise such problems and the translation is direct, preserving both syntactic and semantic structure.

It should be noted that occam-π is not designed to be an execution engine for CSP – that is, that translation from arbitrary CSP systems to occam-π is not always easy or, even, possible[1]. Rather, occam-π is designed as a programming language with concurrency built in as a first-class mechanism, with a semantics directly expressed by CSP. It allows concurrency to be used with the same confidence, ease and overheads as, say, sequential procedures (or method invocation).

3. A LANGUAGE BINDING FOR PROCESS ORIENTED DEVELOPMENT

occam-π is an imperative stateful language built around the concurrency model of Hoare's CSP. Compiler enforced language rules prevent unsynchronised access to

[1]Having said that, there is not much that cannot now be done.

shared resources, so that no data race hazards can happen. Strict aliasing control enables this and provides a simple semantics for assignment.

It extends the classical occam2.1 language [SGS-THOMSON Microelectronics Limited 1995] through the careful blending in of dynamic mechanisms from Milner's π-calculus [Milner 1999] – for example, mobile channels, barriers and processes [Barnes and Welch 2004; Welch and Barnes 2005a; 2005b; 2008] These mobility concepts have much to offer in the modelling of the Santa Claus system, but will be considered in a later paper.

occam-π also extends classical occam through the introduction of *shared* channel-ends (modelled by CSP *interleaving*), barriers (corresponding to *multiway* CSP events) and *extended rendezvous* (simply modelled by adding an event marking the end of the rendezvous). These three mechanisms are employed to simplify the solution presented in this paper.

The occam-π codes were developed with the KRoC [Wood and Welch 1996; Barnes et al. 2010] compiler, run-time system and library – an open-source project originated and hosted at the University of Kent. At present, compiled code is targeted only at i86 platforms (taking full advantage of multicores). Memory overheads (up to 32 bytes per process) and run-time costs (the low tens of nanoseconds per synchronisation) enable millions of processes to be scheduled per processing node and perform useful work [Ritson and Welch 2007]. An interpreted version (the Transterpreter [Jacobsen and Jadud 2004]) is available for almost any target platform, requiring a very tiny memory footprint. Two new compiler projects [Barnes 2006; Sampson 2007; Sampson et al. 2010], targeting all platforms supported by a C compiler, are in development.

3.1 Processes, Sequential Composition and Parallel Composition

A *process* in occam-π is either a *primitive* or a *composition* of processes. A process, at any level, may make local declarations. A process may use its local declarations or anything declared globally (and not hidden) – normal block structuring rules.

It is just as easy, *syntactically*, to compose processes for sequential execution as it is for parallel:

```
SEQ                      PAR
  ...  process A           ...  process A
  ...  process B           ...  process B
  ...  process C           ...  process C
```

In sequential execution, each component sub-process may not start until the previous one has terminated. They may freely share and update global variables.

In parallel execution, all components run concurrently. The construct does not terminate until all its components have terminated. The components may only share global variables for reading: if one sub-process changes a global, the other sub-processes may not even look at it. These rules are statically checked and enforced by the compiler. Note that any component may have its own locals.

3.2 Primitive Processes

There are ten forms of primitive executable process. The first is an assignment: evaluate some expression (RHS) and assign (`:=`) the result to a variable (LHS).

Strong typing rules are enforced. Expression evaluation has no side-effects (as in a functional language). This, together with the strict anti-aliasing enforcement (no entity can have different names in the same scope), means that the semantics of assignment is simple: the assigned variable is set to the assigned value *and nothing else changes*[2].

Five other primitive processes are: channel input and output (Section 3.3), barrier synchronisation (Section 3.6), `SKIP` (which does nothing but terminate – sometimes needed for syntactic place holding), and `STOP` (which does nothing – not even terminate – and gives a concrete manifestation of deadlock, useful for semantic reasoning and model checking). Three more are obtaining time-stamps, setting timeouts and forking processes – but these are not used in this paper.

Finally, process *abstractions* may be named and parametrised:

```
PROC <name> (<parameters>)
  ...  process
:
```

The parameters may be any type, including data (by reference or value) and synchronisations (channel-ends and barriers). The colon above marks the end of the declaration. A named abstraction may be invoked by its name and supplying correctly typed arguments – which is our final syntactic form of executable. Invoked in *sequence* with other processes, they may be thought of as procedures (or methods). Invoked in *parallel* with other processes, they become components of a network whose topology is determined by the synchronisation items they share. The main system developed in this paper (Section 4.3 and Figures 2 and 6) gives an example.

3.3 Channels, Extended Rendezvous and Sharing

Message passing happens through channel communication. Channels have a reading end and a writing end – they are unidirectional. A channel is declared as follows:

```
CHAN <message-type> <name>:
```

The reading end of a channel is denoted by `<name>?` and the writing end by `<name>!`. To write to a channel named `c`:

```
c ! <expression-list>
```

where the message type of the channel and individual expressions in the (semi-colon separated) list must match. Channels are zero-buffered, so the writing process will block until another process, running in parallel with it, executes a read on the other end of the same channel:

```
c ? <variable-list>
```

Here, the message type of the channel and individual variables in the (semi-colon separated) list must match. A reading process will block until another process, running in parallel with it, executes a write on the other end of the same channel.

[2]This is not the case for most other imperative languages – such as C, Java, ...

Only when (or if) both processes reach these respective synchronisation points does the communication happen – whichever process gets there first must wait. After the communication, both processes go their separate (concurrent) ways.

occam-π allows an *extended rendezvous* on channel input:

```
c ?? <variable>
  ...  rendezvous process (must not use c?)
```

This extended input blocks until a message is pending on the `c` channel. For a normal channel input (i.e., a *single* `?`), the writing process would be released at this point. With the above extended channel input (`??`), the release does not happen until after the indented rendezvous block has been completed by the reading process. The writing process is unaware of any such behaviour.

Either of the two ends, or both, can be shared: this is denoted by either `SHARED ?` (shared reading end), `SHARED !` (shared writing end) or `SHARED` (both ends shared) prefixed to the channel declaration. If a channel-end is `SHARED`, it must be claimed for exclusive use:

```
CLAIM c!
  ...  use c! (as many times as you like)
```

The `CLAIM` will *block* if some other process is already holding a claim on this channel-end. Processes wait on a FIFO queue – a different one for each end (if both are shared). A process has exclusive use of the channel-end within the indented block below the `CLAIM`. The claim is automatically released at the end of this block.

It is fairly common that a `CLAIM` block consists of a single line using the claimed channel:

```
CLAIM c!
  c ! 42
```

In such cases, the claim and use may be collapsed to a single line:

```
CLAIM c ! 42
```

All uses of shared channel-ends in this paper fit this pattern.

3.4 Choice

occam-π provides a simple way of waiting for one of a set of events to be offered and, then, making a response. Should more than one of these events become available, an *arbitrary* (i.e., non-deterministic) choice is made. An `ALT`ernative construct is a list of *guarded processes*:

```
ALT
  <guard>
    <process>
  ...
  <guard>
    <process>
```

The list order does not matter. The guards are the *waited-for* events – currently, only input processes (simple/extended, on offer when a message is pending), timeouts (on offer when expired) and `SKIP`s (always on offer) are allowed.

If control of the choice is needed should more than one event be on offer, a prioritised version (PRI ALT) version is available. This resolves the choice in favour of the first one listed, so that the ordering of the guarded processes does matter in this case.

Used with a SKIP as its final guard, a PRI ALT lets us *poll* channels for input and get on with something else if none is available. It also gives a way to force a process away from servicing an always-busy channel. The *unfairness* built in to the PRI ALT curiously, and simply, gives a way to guarantee *fairness* is servicing events. For example, a two-input multiplexor can be made fair by unrolling the loop body twice (in SEQuence), changing both ALTs into PRI ALTs and switching the order of the second one. A PRI ALT is also exactly suited to Santa's duty to go with the reindeer in case both the reindeer and a party of elves are knocking on Santa's door.

3.5 Replicators

The SEQ, PAR and ALT constructs may be replicated. Suppose XXX is one of these three keywords. Then:

```
XXX i = start FOR n
  <process.which.may.use.i>
```

is short-hand for:

```
XXX
  <process.with.i.replaced.by.start>
  <process.with.i.replaced.by.(start + 1)>
  ...
  <process.with.i.replaced.by.(start + (n - 1))>
```

The replicated SEQ corresponds to a traditional for-loop (with guaranteed termination). The replicated PAR sets up regular network topologies. In a replicated ALT, the <process.which.may.use.i> must be a guarded process and the construct waits for, and chooses between, an array of events.

3.6 Barriers

The last concept needed for this paper is the *barrier*. A barrier is multiway synchronisation point. No process can proceed past the barrier until every process enrolled on the barrier has reached it. The syntax for declaring and enrolling processes on a barrier is as follows:

```
BARRIER <barrier-name>:

PAR ENROLL <barrier-name>
  ...  all processes here are enrolled
```

Synchronising on a barrier is the last occam-π primitive we need:

```
SYNC <barrier-name>
```

4. AN OCCAM-π IMPLEMENTATION

The Santa Claus system has three kinds of component: Santa, reindeer and elf. Their states are outlined in on-line Appendix A, along with the reports we require them to make as they cycle through them.

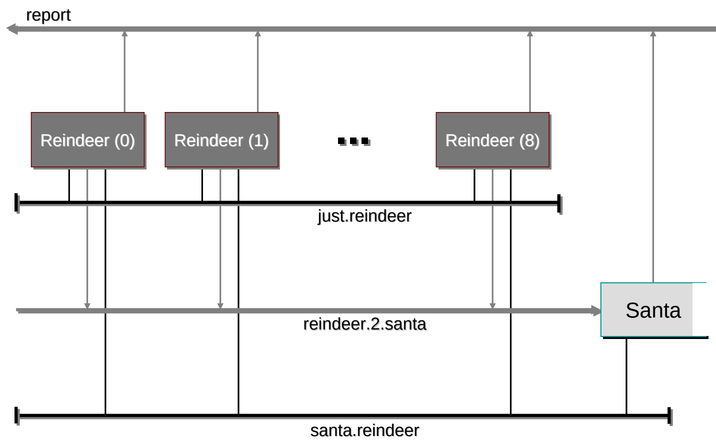


Fig. 2. Santa and the reindeer.

The Santa Claus story has near symmetry. The actions performed by reindeer and elves are much the same: they do things on their own for a while (holidaying, toy development), then rendezvous with Santa (9 out of 9, 3 out of 10) and work with him (delivering toys, consulting), then repeat. The *partial* nature of the Santa-elves rendezvous, however, requires special treatment compared with the *full* rendezvous between Santa and all the reindeer (which can be handled by a primitive CSP/occam-π multiway event). Symmetry could be restored by declining that primitive. Then, we would need only one type of process that could play the role of either a reindeer or elf – and Santa's processing of either would be the same. However, in this presentation, we will work with separate logic and explore the different mechanisms.

4.1 Santa and the Reindeer

Figure 2 shows Santa and the reindeer processes connected in a network of channels and barriers.

There is a `report` channel, whose writing end is *shared* by all the processes and through which they report their life stories. This channel is external to the system and may be used to animate a display of what is happening inside the system. It could also be used to control a machine whose components are modelled by the Santa, reindeer and elf processes. It may be more appropriate to have separate (parallel) report channels – one for each component of the system. However, the semantics of the system are such that there is no difference between these two views. Each component makes its reports independently, identifying itself in each report. Those identifying tags partition the set of reports into component-unique subsets that can be viewed as reports on separate channels. For synchronisation analysis, all we need to know is that reports can be made at any time and will never be blocked by the actions (or inactions) of other components within the system. We may assume that the environment of the system will always always accept these messages.

There is a `just.reindeer` barrier, on which the reindeer wait for each other to return from holiday. There is a `santa.reindeer` barrier, on which the reindeer wait for Santa to harness all of them before starting the sleigh run. This is also used to wait for Santa to bring them all home after delivering toys. Finally, there is a `reindeer.2.santa` channel, whose writing end is shared by the reindeer. When all the reindeer are back from holiday, they report in to Santa (for harnessing) by sending their names through this channel – they will, of course, be blocked at this point if Santa is consulting with elves. The reindeer also use this channel to synchronise with Santa so that he can unharness them after delivering toys.

Here is the reindeer process:

```
PROC reindeer (VAL INT id, BARRIER just.reindeer, santa.reindeer,
               SHARED CHAN INT to.santa!, SHARED CHAN REINDEER.MSG report!)
  WHILE TRUE
    SEQ
      CLAIM report ! holiday; id      -- "I'm on holiday" + id
      random.wait (HOLIDAY.TIME)      -- sleep for random amount of time
      CLAIM report ! deer.ready; id   -- "I'm back from holiday" + id
      SYNC just.reindeer              -- wait for all deer to return
      CLAIM to.santa ! id             -- send id and get harnessed
      SYNC santa.reindeer             -- wait for others to be harnessed
      CLAIM report ! deliver; id      -- "I'm delivering toys" + id
      SYNC santa.reindeer             -- until Santa takes us all home
      CLAIM report ! deer.done; id    -- "I'm back from sleigh run" + id
      CLAIM to.santa ! id             -- get unharnessed
:
```

Here is the part of the header and body of the Santa process that deals with the reindeer:

```
PROC santa (CHAN INT from.reindeer?, BARRIER santa.reindeer,
            ...  elf connections
            SHARED CHAN SANTA.MSG report!)
  WHILE TRUE
    PRI ALT                           -- wait for reindeer or elves
      ...  deal with the reindeer
      ...  deal with the elves
:
```

Each of the above *dealing* processes is prefixed by a signal indicating, respectively, that either all `N.REINDEER` (9) reindeer are back from holiday or that a group of `G.ELVES` (3) elves, out of `N.ELVES` (10), is ready to consult. The `PRI ALT` ensures that if there are both reindeer and elves signalling, Santa will choose the reindeer.

Here is Santa's code dealing with the reindeer. The wake-up signal is just the message from *one* (any one) of the gathered reindeer, giving its name. This is followed by the messages from all the other reindeer – in some arbitrary order:

```
{{{  deal with the reindeer
INT id:
from.reindeer ? id                    -- the first reindeer is here
  SEQ
    CLAIM report ! reindeer.ready     -- "Ho, Ho, Ho, reindeer are here"
    CLAIM report ! harness; id        -- "Harnessing reindeer " + id
    SEQ i = 0 FOR N.REINDEER - 1      -- for the remaining deer
      SEQ
        from.reindeer ? id            -- receive their id
        CLAIM report ! harness; id    -- "Harnessing reindeer " + id
    CLAIM report ! mush.mush          -- "Mush Mush"
    SYNC santa.reindeer               -- tell reindeer all are harnessed
    random.wait (DELIVERY.TIME)       -- deliver toys for some random time
    CLAIM report ! woah               -- "Whoa ... let's go home"
    SYNC santa.reindeer               -- signal everyone to return home
    SEQ i = 0 FOR N.REINDEER          -- for each deer
      from.reindeer ?? id             -- receive their id
        CLAIM report ! unharness; id  -- "Unharnessing reindeer " + id
}}}
```

The reindeer, once they have reported to Santa for unharnessing, loop around to announce they are on holiday again. Santa does the unharnessing and reports it, holding each reindeer in an extended rendezvous whilst that happens. This ensures that the unharnessing report happens before the holiday report. Note that this is not necessary for the harnessing reports, since the reindeer wait for the whole team to be harnessed (`santa.reindeer`) before their next report (that they are delivering toys).

There are other *happens-before* relationships enforced by the above `santa` and `reindeer` processes:

—all reindeer must have reported their return from holiday before Santa is woken up to announce that the reindeer are here – enforced by the `SYNC` on `just.reindeer` in the `reindeer` process;

—all reindeer must be harnessed before Santa says "Mush Mush" – enforced by sequential code in the `santa` process;

—Santa says "Mush Mush" before any reindeer reports it is delivering toys – enforced by the first `SYNC` on `santa.reindeer` in the `reindeer` and `santa` processes;

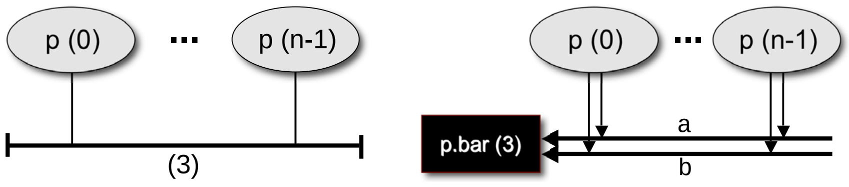
—Santa says "Whoa" before any reindeer reports it is back from delivering toys – enforced by the second `SYNC` on `santa.reindeer` in the `reindeer` and `santa` processes.

Later in this paper, we show how these relationships can be formally specified and verified.

4.2 Santa and the Elves

Neither CSP nor occam-π have primitives for the *partial* barrier synchronisation required for this part of the system. So, we have to model the ideas with special processes.

4.2.1 *Partial Barriers.* A partial barrier synchronises any x out of y (other) processes, where $0 < x \leq y$. It is managed by a simple protocol, comprising two channel communications per synchronising process to complete each partial barrier. The first makes the offer to synchronise and the second forces it to wait for enough offers to have been made. These communications are received and managed by a simple process that counts down the offers and, then, accepts the release signals:



(a) n processes enrolled on a *partial* barrier (any process synchronising must wait for 2 others to synchronise).

(b) implementation of a *partial* barrier (using current occam-π mechanisms).

Fig. 3. A partial barrier.

```
PROC p.bar (VAL INT x, CHAN BOOL a?, b?)
  WHILE TRUE
    SEQ
      SEQ i = 0 FOR x    -- gather in the right number of offers
        BOOL any:
        a ? any
      SEQ i = 0 FOR x    -- all offers received: let everyone go
        BOOL any:
        b ? any
:
```

The partial-barrier-synchronising processes plug into shared writing ends of the channels connecting to `p.bar`. To synchronise, they run:

```
PROC sync (SHARED CHAN BOOL a!, b!)
  SEQ
    CLAIM a ! TRUE    -- offer to synchronise
    CLAIM b ! TRUE    -- wait for enough offers
:
```

This `sync` process completes when, and only when, it is one of x processes (out of y) engaging in the partial barrier (i.e., plugged into `p.bar`).

This construction has an obvious visual representation. Diagram (a) in Figure 3 shows n processes enrolled on a partial barrier requiring 3 synchronisation offers to complete. Diagram (b) shows the implementation given in this subsection.

4.2.2 *Extended Partial Barriers.* The `p.bar` alone is not sufficient to solve the elves waiting problem when they want to see Santa. If they `sync` as above on an `p.bar` set to 3, when three elves get through the barrier they will try to see Santa (who may be away with the reindeer). As soon as that group of elves are through the barrier, another group (out of the 7 left) can come along and also complete the barrier. There will now be two groups of three elves trying to see Santa – this is not allowed!

A common idiom with barriers is to schedule some special code to be run on completion of the barrier *but before* the processes engaging on the barrier are released. This can be achieved by generating a signal from the process managing the barrier to trigger that code: To do this, we extend `p.bar` to become the `xp.bar` process by adding a signal channel (`CHAN BOOL ping!`) to its parameter list and outputting

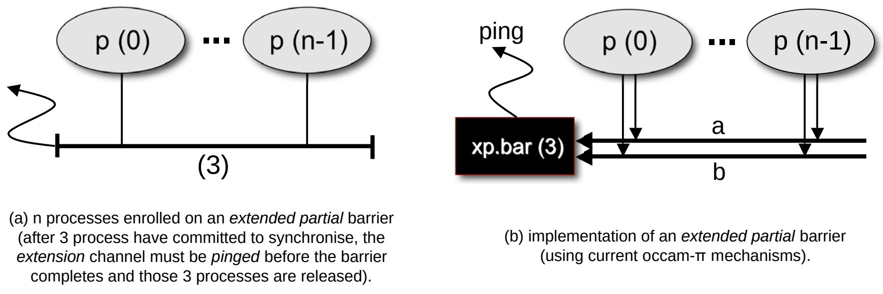

p (0)
p (n-1)
(3)
(a) n processes enrolled on an *extended partial* barrier (after 3 process have committed to synchronise, the *extension* channel must be *pinged* before the barrier completes and those 3 processes are released).
ping
p (0)
p (n-1)
xp.bar (3)
a
b
(b) implementation of an *extended partial* barrier (using current occam-π mechanisms).

Fig. 4. An extended partial barrier.

TRUE on ping! between its two replicated SEQs. The code implementing this is in on-line Appendix C.

Now, all that is needed is to wrap the code to be triggered in a process that waits for the *ping* signal and runs it. To ensure that this code completes before the triggering barrier, simply run it inside an extended rendezvous:

```
BOOL any:
ping ?? any
  ...  process triggered by barrier
```

Sometimes that condition is not necessary – all that is needed is that the special code is triggered. This is the case for our system.

Diagram (a) in Figure 4 visualises n processes enrolled on an extended partial barrier requiring 3 synchronisation offers to complete. Diagram (b) shows the implementation given in this subsection.

4.2.3 *Elves and Santa.* Figure 5 shows Santa and the elf processes connected in a network of channels and partial barriers.

There is the same `report` channel as shown in Figure 2. This is used by Santa and the elves for the same reasons as before. There is a `just.elves` extended partial barrier of size three, on which elves wait for two others to want to see Santa. When three are present, a (*knock*) on Santa's door is generated, via the extension channel belonging to the barrier. When Santa accepts that knock, the barrier completes and the three elves can consult with Santa. This corresponds to the `just.reindeer` barrier from Figure 2.

There is a `santa.elves` (non-extended) partial barrier of size four. This is used by each group of three elves to wait for santa to greet *all* of them before the consultation starts. This is also used by the elves to wait for Santa to conclude the consultation. Only Santa and the current consulting group of elves will ever try to use this partial barrier. This corresponds to the `santa.reindeer` barrier from Figure 2.

There is an `elves.2.santa` channel, whose writing end is shared by the elves. When a group of three elves have assembled, they introduce themselves through this channel. They will not be blocked at this point, since Santa is expecting them (having been awoken by the *knock* on his door from `xp.bar`). This channel is also

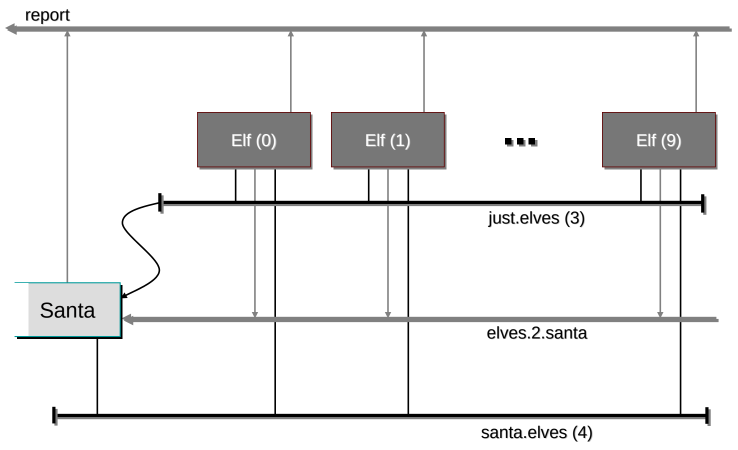


Fig. 5. Santa and the elves.

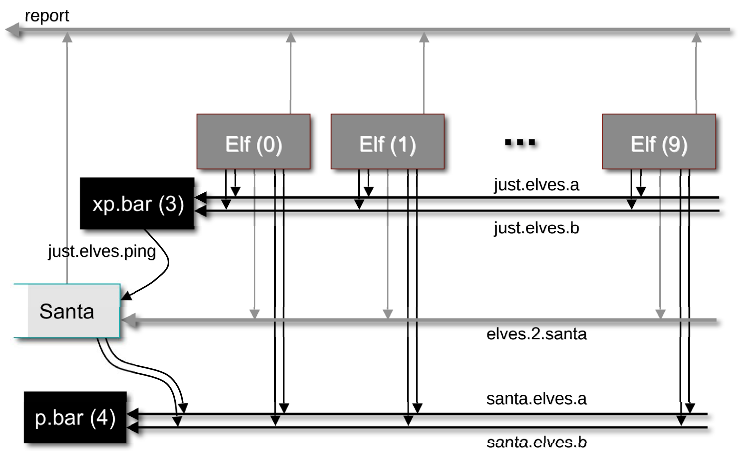


Fig. 6. Santa and the elves (transformed).

used to say goodbye to Santa when the consultation is finished. Since occam-π does not (currently) support partial barrier connectors, the transformations defined by Figures 3 and 4 are applied to yield (the directly implementable) Figure 6.

In terms of Figure 6, here is the elf process:

```
PROC elf (VAL INT id,  SHARED CHAN BOOL just.elves.a!, just.elves.b!,
          SHARED CHAN BOOL santa.elves.a!, santa.elves.b!,
          SHARED CHAN INT to.santa!, SHARED CHAN ELF.MSG report!)
  WHILE TRUE
    SEQ
      CLAIM report ! working; id             -- "I'm working" + id
      random.wait (WORKING.TIME)             -- until I have a problem
      CLAIM report ! elf.ready; id           -- "I want to see Santa" + id
      sync (just.elves.a!, just.elves.b!)    -- wait for two other elves
      CLAIM to.santa ! id                    -- say hello to Santa
      sync (santa.elves.a!, santa.elves.b!)  -- wait for others to say hello
      CLAIM report ! consult; id             -- "Consulting Santa" + id
      sync (santa.elves.a!, santa.elves.b!)  -- until Santa has had enough
      CLAIM report ! elf.done; id            -- "I'm done consulting" + id
      CLAIM to.santa ! id                    -- say goodbye to Santa
:
```

Completing the declaration of the Santa process header from Section 4.1, here are the channels for communicating with elves that were omitted:

```
        CHAN BOOL just.elves.ping?, CHAN INT from.elves?,
        SHARED CHAN BOOL santa.elves.a!, santa.elves.b!,
```

Recall that, for the elves, Santa is awoken by a *ping* from the extension channel of the partial barrier (and not by a communication from one of the elves):

```
  {{{  deal with the elves
  BOOL any:
  just.elves.ping ? any                    -- a party of elves is at door
    SEQ
      CLAIM report ! elves.ready           -- "Ho, Ho, Ho, elves are here"
      SEQ i = 0 FOR G.ELVES                -- for each elf in party
        INT id:                            -- (G.ELVES is size of party)
        SEQ
          from.elves ? id                  -- receive elf id
          CLAIM report ! greet; id         -- "Hello elf " + id
      CLAIM report ! consulting            -- "Consulting with elves"
      sync (santa.elves.a!, santa.elves.b!)  -- tell elf party all are here
      random.wait(CONSULTATION.TIME)       -- consult for a random time
      CLAIM report ! santa.done            -- "Ok, all done - thanks!"
      sync (santa.elves.a!, santa.elves.b!)  -- tell elves consultancy over
      SEQ i = 0 FOR G.ELVES                -- for each elf in party
        INT id:
        from.elves ?? id                   -- receive elf id
          CLAIM report ! goodbye; id       -- "Goodbye elf " + id
  }}}
```

Each elf is held (in an extended rendezvous) whilst Santa says goodbye for the same reason that Santa held each reindeer whilst unharnessing: so that the "goodbye" report happens before the elf can get back to work and report "working". There are other *happens-before* relationships enforced by the above `santa` and `elf` processes:

—three elves must have reported that they want to see Santa before Santa is woken up and announces that the elf party is here – enforced by the extended partial barrier, `just.elves`, between the `elf` processes (moderated by the `ping` wake-up call to Santa);

—the elf party must be greeted before Santa says "Consulting with elves" – enforced by sequential code in the `santa` process;

—Santa says “Consulting with elves” before any elf reports that it is consulting – enforced by the first partial barrier sync on `santa.elves` in `elf` and `santa`;

—Santa says “all done” before any elf reports its consultancy session is over – enforced by the second partial barrier sync on `santa.elves` in `elf` and `santa`.

4.3 The Santa Claus System

The Santa Claus system consists of everything shown in Figures 2 and 6. The `report` channel is left open (an external parameter) with all other events (channels and barriers) hidden (local).

```
PROC santa.system (SHARED CHAN MESSAGE report!)
  BARRIER just.reindeer, santa.reindeer:
  SHARED ! CHAN INT reindeer.2.santa:
  SHARED ! CHAN BOOL just.elves.a:           -- extended partial barrier
  SHARED ! CHAN BOOL just.elves.b:           --   channels for just.elves
  CHAN BOOL just.elves.ping:                 --   (extension channel)
  SHARED ! CHAN BOOL santa.elves.a:          -- partial barrier channels
  SHARED ! CHAN BOOL santa.elves.b:          --   for santa.elves
  SHARED ! CHAN INT elves.2.santa:
  PAR
    PAR ENROLL santa.reindeer
      santa (reindeer.2.santa?, santa.reindeer,
             just.elves.ping?, elves.2.santa?,
             santa.elves.a!, santa.elves.b!, report!)
      PAR i = 0 FOR N.REINDEER ENROLL just.reindeer, santa.reindeer
        reindeer (i, just.reindeer, santa.reindeer,
                  reindeer.2.santa!, report!)
    PAR i = 0 FOR N.ELVES
      elf (i, just.elves.a!, just.elves.b!,
           santa.elves.a!, santa.elves.b!, elves.2.santa!, report!)
    xp.bar (G.ELVES, just.elves.a?, just.elves.b?, just.elves.ping!)
    p.bar (G.ELVES + 1, santa.elves.a?, santa.elves.b?)
:
```

The complexity of the implementation depends on the number of reports required and how strictly they must conform to *happens-before* relations. For instance, an elf has two waiting states it could report: a wait to get into the *front* group (or *waiting room*) that will be next to see Santa and, then, a wait to see Santa (who may be away delivering toys).

In the model built in this section, an elf only reports when it wants to see Santa. In Section 6.1, we extend the elf implementation to report both waiting states. We require three elf reports that they have made the front group *before* Santa reports he has been awakened by them. Initially, the arrival of the third elf in the front group automatically triggers a knock on Santa's door and, if Santa is sleeping, his awakening. The third elf's report (that it is in the front group) and Santa's report (that he has been woken by elves) proceed in parallel and could happen in either order. The correct ordering is later enforced and verified, but extra synchronisations (and complexity) are needed.

5. VERIFYING PROPERTIES OF THE SOLUTION

The Santa Claus Problem can be viewed as an exercise in programming concurrent control logic for safety critical systems. The reliability of such a system is the most important issue. The software should be formally verified to assure that it is free from deadlock and livelock, free from race conditions, and that the ordering of control signals generated by the software does not violate any specified constraints.

A number of obstacles complicate formal verification of code. Firstly, a translation from the code into the formal model used for verification is necessary (or in the case where formal verification is performed before the implementation, a translation from the formal model to the programming language of choice). This is potentially a challenging task, especially if the language and the formalism have little in common. The greater the gap, the greater the risk of introducing bugs becomes. Secondly, todays verification tools are rarely capable of checking a complete program without any user support.

Therefore, we believe that the best solution to producing correct (formally verified) code is by choosing a formalism and a language that are related and a verification tool that can assist in 'computer driven verification', that is, certain parts of the code can be verified, and the rest must be formally argued by the programmer.

The solution to the Santa Claus Problem presented in the previous section is expressed in the process oriented language occam-π, a language **based** on the the formal process algebra CSP as well as the π-calculus [Milner 1999]. A formalisation in CSP is extremely close to the actual occam-π code. This relationship is crucial in reducing the number of errors when translating between the language and the formalism or vice versa.

To verify the specification in CSP, we utilise the FDR [Formal Systems (Europe) Ltd. 1998] tool. FDR can check a number of properties about a CSP specification, for example: freedom from deadlock and livelock. It can also verify process *refinement* and *equivalence* under three semantic models of growing strength (*traces*, *failures*, and *failures-divergences*.)

5.1 The CSP Model

In this section, we translate the occam-π model of the Santa Claus system given in Section 4 into CSP. The CSP structure (syntax and semantics) directly matches the occam-π code and could, therefore, have been presented first.

However, occam-π does not pretend to be an implementation of CSP and translating in the other direction is harder. Some CSP features have large overheads (for the execution mechanisms currently known and in the occam-π run-time kernel) and are not yet supported – for example, external choice over arbitrary multiway events. General CSP systems *can* be implemented in occam-π, but they have to be transformed first into equivalent forms that do have direct representation [McEwan 2006]. Although this can always be done, it can add much complexity.

occam-π is a *programming language* most of whose concurrency mechanisms have a direct model in CSP (and, therefore, semantics). We prefer, normally, to design within the patterns allowed by occam-π, knowing that our systems are directly and efficiently executable and that formal reasoning and/or model checking can be directly applied.

Some occam-π mechanisms are not addressed by CSP – such as timing and priorities. So, the random delays in the occam-π model (e.g. when a reindeer is on holiday) are treated as SKIPs in the CSP and the PRI ALT in the santa process is mapped to a plain external choice. Shared channel-ends are modelled by the sharing processes *interleaving* on them. The occam-π CLAIM mechanism is a refinement of this interleaving, since processes trying to use them are fairly *queued* (rather than having the *arbitrary* access allowed by interleaving).

The occam-π extended rendezvous is simply modelled in CSP by introducing an *acknowledgement* event for the channel communication. Both processes must engage on that acknowledgement following each communication – immediately by the sender but after the rendezvous block by the receiver.

We present the CSP using the machine-readable syntax, CSP-M, accepted by the FDR [Formal Systems (Europe) Ltd. 1998] model checking tool. The CSP constants, types and events used by the system are given in the on-line Appendix D. This appendix also contains the CSP definitions of the partial barrier processes (P_BAR, XP_BAR), the reindeer process (REINDEER) and the complete Santa Claus system (SYSTEM). Here, we present the CSP versions of just the elf and Santa processes to demonstrate the relationship between occam-π executable code and CSP verifiable expressions.

5.2 Elves

We define the elf process directly from the occam-π declaration in Section 4.2.3:

```
ELF (id) =
  report ! working.id -> RANDOM_TIMEOUT (WORKING_TIME) ;
  report ! elfReady.id -> just_elves_a -> just_elves_b ->
  elves_2_santa ! id -> santa_elves_a -> santa_elves_b ->
  report ! consult.id -> santa_elves_a -> santa_elves_b ->
  report ! elfDone.id -> elves_2_santa ! id ->
  elves_2_santa_ack -> ELF (id)
```

where, as mentioned above:

```
RANDOM_TIMEOUT (WORKING_TIME) = SKIP
```

Note: for simplicity, the above process has been bound directly to its channels. If needed (and it is not for this system), instances of this process could be bound to other channels through *channel renaming*. We will define the other processes similarly bound to their channels.

5.3 Santa

Here is the Santa process, following Sections 4.1 and 4.2.3:

```
SANTA =
  (reindeer_2_santa ? id ->
    report ! allReindeer.0 -> report ! harness.id ->
    (; x:<0..(G_REINDEER - 2)> @
      (reindeer_2_santa ? id -> report ! harness.id -> SKIP)) ;
    report ! mushMush.0 -> santa_reindeer ->
    RANDOM_TIMEOUT (DELIVERY_TIME) ;
```

```
      report ! woah.0 -> santa_reindeer ->
      (; x:<0..(G_REINDEER - 1)> @
        (reindeer_2_santa ? id -> report ! unharness.id ->
         reindeer_2_santa_ack -> SKIP)) ;
      SANTA
    )
    []
    (just_elves_ping ->
      report ! threeElves.0 ->
      (; x:<0..(G_ELVES - 1)> @
        (elves_2_santa ? id -> report ! greet.id -> SKIP)) ;
      report ! consulting.0 -> santa_elves_a -> santa_elves_b ->
      RANDOM_TIMEOUT (CONSULTATION_TIME) ;
      report ! done.0 -> santa_elves_a -> santa_elves_b ->
      (; x:<0..(G_ELVES - 1)> @
        (elves_2_santa ? id -> report ! goodbye.id ->
         elves_2_santa_ack -> SKIP)) ;
      SANTA
    )
```

5.4 Deadlock and Livelock

Deadlock freedom is a crucial property that all systems (probably) should have. The FDR tool contains an option for checking the presence or the absence of deadlock; it is as simple as loading the desired specification and clicking the 'deadlock' button. In practice, we might encounter one of the problems described earlier: if the model is large, FDR might either run for a very long time, or simply run out of memory (exploring the state space needed for the model) and terminate without an answer.

We did encounter this problem when asking FDR to analyse the full model with 10 elves, 9 reindeer, and one Santa. This is where fully automated verification becomes *machine assistance* to further reasoning.

Each `report` made by an elf, reindeer or Santa adds one state to that process. Each reindeer and elf cycle through 4 reports each. Santa has 21 different reports. With nine reindeer, ten elves and one Santa, the *potential* state space increase arising from the reports is over eight trillion. The *actual* increase will be less than this, since large areas of state space will be barred because of the other internal synchronisations. Nevertheless, that still opens up too much space to analyse.

We need to reduce the state space in the system given to FDR. The `report` channel is the only one not hidden within `SYSTEM` (see Appendix D). None of the `SYSTEM` sub-processes synchronise with each other to make a `report` – they all interleave on its use. Therefore, no `SYSTEM` process can ever be blocked (for internal reasons) trying to make a `report`. For deadlock analysis of `SYSTEM`, this `report` channel is irrelevant and may be removed. Resubmitting `SYSTEM` (without any reports), FDR confirms that this system is deadlock free. We can now deduce that the original `SYSTEM` (with all the reports) is also deadlock free.

To prove the absence of livelock, we click the 'livelock' button. FDR immediately reports that the version of `SYSTEM` (without any reports) is **not** livelock free. This is because we removed all external signalling, but left the system still running. That **is** livelock!

To deal with this, a third version of `SYSTEM` is produced that leaves in just *two* reports, one in each branch of the alternation in the Santa process. We now have external reports whenever a group of elves or reindeer wake up Santa. FDR confirms that the system is livelock-free. There can be no infinite sequence of purely internal actions. Putting back all the reports merely adds external actions. Therefore, the original `SYSTEM` is also livelock-free.

This is a major achievement in terms of proving properties for safety critical software. For example, it would be catastrophic if the system controlling the flight correction system of the B2 plane deadlocked. (The B2 plane is aerodynamically unstable on all three axes and require constant flight correction; a task of which no human is capable, thus it must be done by computerised fly-by-wire systems [Wikipedia 2007]). This would most likely cause the plane to lose control and crash.

We have now succeeded in proving two of the three types of properties that we wished to reason about, namely the absence of deadlock and livelock. What remains is to reason about the ordering of output signals, which we will do in the following section.

5.5 Event Ordering and Synchronisation

In the previous section we illustrated the use of the FDR tool to verify that the implementation does not livelock or deadlock. Another issue is of equal importance exists, namely the ordering of any control signals it generates.

In our solution of the Santa Claus problem, all processes share a `report` channel. Considering these reports as control signals, we now concern ourselves with ensuring that our control logic does not generate incorrect sequences of these signals.

The idea of a shared reporting channel over which control messages for an embedded system are serialised is not very realistic. In such a system, each internal component may be wired directly to the device it controls, so that the signals may travel in parallel. In fact, CSP makes no semantic distinction between a realisation of the `report` channel as a *shared* channel (down which reports are serialised) and a *parallel array* of channels indexed by `ReportTag` – CSP semantics serialise *all* events. The occam-π implementation (Section 4) defines a `SHARED` report channel. This could trivially be changed to an array of separate reporting channels, with no change to the CSP formalisation.

Simply observing serialised reports from `SYSTEM` may reveal out-of-order reports. Of course, if none are observed, this does not prove that such things may not happen. However, we can capture assertions about trace ordering in CSP in such a way that their adherence by a system can be verified. Here is one such (informal) assertion:

> Santa never says "Ho Ho Ho ... Some elves are here" **until at least** three elves have reported that they are ready to consult.

To check such assertions formally, we turn to the FDR tool once again. First we devise a process that performs the check and causes deadlock if the check fails. Then, we add this checking process to the system and ask FDR if it is still deadlock-free. If that passes, we know that the assertion is always honoured. For the given assertion, we need a checker that:

—accepts and counts inputs on the `report` channel (`report ? x.y`);

—ignores reports whose message type (`x`) is not `elfReady` or `threeElves`;

—adds one to the count if the message type is `elfReady`;

—`STOP`s if the message type is `threeElves` and the count is less than three – otherwise removes three from the count.

Here it is:

```
CHECK_A (n) =
  report ? x.y ->
    if x == elfReady then
      if n > N_ELVES then STOP else CHECK_A (n+1)
    else if x == threeElves then
      if n < 3 then STOP else CHECK_A (n-3)
    else
      CHECK_A (n)
```

where `STOP` is a process that refuses to engage in any activity – not even termination. It represents a deadlocked process.

FDR cannot analyse any process containing *unbounded* recursion. The test whose falsity triggers `CHECK_A (n+1)` in the preceding script will always be false when this checker is run in parallel with `SYSTEM`. However, this extra test bounds the recursion; so far as the FDR analyser is concerned, we need it! The other test whose falsity triggers `CHECK_A (n-3)` later in the script is, of course, essential for the purpose of the check.

Now, add this process into the system, initialising its count to zero:

```
CHECK_A_SYSTEM = SYSTEM [| {| report |} |] CHECK_A (0)
```

The checking process and the original system synchronise on the `report` channel in the above. If `SYSTEM` ever generates a `threeElves` message without there being at least three outstanding `elfReady` messages, the `CHECK_A (0)` process stops. Any further reports from `SYSTEM` will be blocked. Since Santa has to generate a report (at least) once per cycle, Santa will stop. The reindeer and elves communicate with Santa (at least) once per cycle, so they will all stop. The two partial barrier processes also communicate with Santa (at least) once per cycle, so they will all stop. Hence, `CHECK_A_SYSTEM` will deadlock. FDR says `CHECK_A_SYSTEM` will not deadlock. Therefore, the assertion never fails.

6. REPORTING MORE DETAIL

There are more states in the `SYSTEM` than are currently being reported.

6.1 Santa's Waiting Room

An interesting example concerns the assembly of elves into groups of three. There are two stages that each elf goes through, represented by the events `just_elves_a` and `just_elves_b`. We may imagine the state in between these events represents an elf being in Santa's *waiting room*, a room that can hold only three of them (a property enforced by the logic within `XP_BAR` process). To observe these elf states, we add another report to the elf process (Section 5.2):

```
ELF (id) =
  ...
  just_elves_a -> report ! elfWaiting.id -> just_elves_b ->
  ...
```

Let us verify that the `elfWaiting` reports obey the same constraints as `elfReady` with respect to the `threeElves` reports from Santa:

> Santa never says "Ho Ho Ho ... Some elves are here" **until at least** three elves have reported that they are in the waiting room.

Similar to `CHECK_A` in Section 5.5, we define:

```
CHECK_B (n) =
  report ? x.y ->
    if x == elfWaiting then
      if n > N_ELVES then STOP else CHECK_B (n+1)
    else if x == threeElves then
      if n < 3 then STOP else CHECK_B (n-3)
    else
      CHECK_B (n)
```

Fortunately, when FDR is asked to consider:

```
CHECK_B_SYSTEM = SYSTEM [| {| report |} |] CHECK_B (0)
```

its deadlock freedom check fails. The event trace leading to deadlock reported by FDR shows that a Santa `threeElves` report can occur before *any* `elfWaiting` report.

In `CHECK_A_SYSTEM`, a similar problem does not arise because Santa's `threeElves` report requires a prior `just_elves_ping`, which requires the `XP_BAR` process to have received three `just_elves_a` events (each of which requires an `elfReady` report to have been made).

In the `CHECK_B_SYSTEM`, deadlock does arise. Santa's `threeElves` report still requires a prior `just_elves_ping`, which requires the `XP_BAR` process to have received three `just_elves_a` events. None of these requires any `elfWaiting` report to have been made. FDR observes, therefore, that `threeElves` may occur first – provoking `STOP` in `CHECK_B` and deadlock in `CHECK_B_SYSTEM`. This needs a little attention.

6.2 A Better Waiting Room

We need to modify the system so that `CHECK_B` does not cause deadlock. We need to enforce the `elfWaiting` reports to be made *before* `just_elves_ping`. Once we know we have to do this, it is easy – simply add an *acknowledgement* event for the `just_elves_a` event and report `elfWaiting` in between. This requires modifications to both the `ELF` process and to `XP_BAR` (Appendix D.2). Because the latter now has behaviour beyond an extended partial barrier, we rename it as `WR` (for *waiting room*):

```
WR (n) =
  (; x:<1..n> @ (just_elves_a -> just_elves_a_ack -> SKIP)) ;
  just_elves_ping -> (; x:<1..n> @ (just_elves_b -> SKIP)) ; WR (n)
```

The elf process becomes:

```
ELF (id) =
  ...
  just_elves_a -> report ! elfWaiting.id ->
  just_elves_a_ack -> just_elves_b ->
  ...
```

Now, rebuild `SYSTEM` as `SYSTEM_WR`, substituting `WR` for `XP_BAR`. Then, ask FDR to analyse:

```
CHECK_B_SYSTEM_WR = SYSTEM_WR [| {| report |} |] CHECK_B (0)
```

and it reports this deadlock free. Therefore, the assertion from Section 6.1 is verified for this new system.

6.3 An Even Better Waiting Room

The waiting room only has room to hold *three* elves at a time. We should be able to strengthen our assertion concerning the elf reports that they are in the waiting room and Santa's welcome report as follows:

> Santa never says "Ho Ho Ho ... Some elves are here" **until exactly** three elves have reported that they are in the waiting room.

Here is the necessary check:

```
CHECK_C (n) =
  report ? x.y ->
    if x == elfWaiting then
      if n >= 3 then STOP else CHECK_C (n+1)
    else if x == threeElves then
      if n < 3 then STOP else CHECK_C (0)
    else
      CHECK_C (n)
```

However, when FDR analyses

```
CHECK_C_SYSTEM_WR = SYSTEM_WR [| {| report |} |] CHECK_C (0)
```

it reports deadlock, with the triggering trace showing *four* `elfWaiting` reports and no `threeElves`. The problem is that, although the waiting room does not allow four elves to be present, as soon it wakes up Santa (`just_elves_ping`) it releases its three elves (`just_elves_b`) and recurses to allow the next group of elves to assemble (`just_elves_a`). The arrival of the first elf in this next group and its reporting of that fact proceeds in parallel with Santa's report welcoming the previous group (`threeElves`). These two reports may happen in either order – hence the violation.

To enforce our required constraint on the ordering of these reports is also easy. Simply delay the recursion of the waiting room until Santa has made his welcoming report – no elf wanting to enter can do so until that recursion happens. To enforce this delay, modify the waiting room process to offer a second `just_elves_ping` to Santa, which he does not accept until he has made that welcoming report.

```
WR (n) =
  (; x:<1..n> @ (just_elves_a -> just_elves_a_ack -> SKIP)) ;
```

```
    just_elves_ping -> (; x:<1..n> @ (just_elves_b -> SKIP)) ;
    just_elves_ping -> WR (n)
```

Santa's response to being woken up by a consulting group of elves needs to change:

```
  SANTA =
    (reindeer_2_santa ? id -> ... )
    []
    (just_elves_ping ->
      report ! threeElves.0 ->
      (; x:<0..(G_ELVES - 1)> @
        (elves_2_santa ? id -> report ! greet.id -> SKIP)) ;
      just_elves_ping -> report ! consulting.0 -> ...
    )
```

For verifying `CHECK_C`, the second `just_elves_ping` could have been made immediately following the `threeElves` report. We made `SANTA` issue his greeting reports to the arriving elves first to ensure these reports are completed before any new elves report they are in the waiting room – but that is a different issue.

Now, when FDR is asked to verify deadlock freedom of `CHECK_C_SYSTEM_WR`, it does. Sections 4.1 and 4.2.3 list several other ordering constraints. These can all be verified in the same way.

6.4 Event Ordering through Refinement

Previously, we have demonstrated verification through *deadlock* checking; another approach uses *refinement* checking. FDR analyses CSP processes with respect to three semantic models: *traces*, *failures* and *failures-divergences*. A process *trace* is a finite ordered sequence of events that it can perform. A process *failure* is a *trace* paired with a *refusal* set (which is a set of events that, when offered to the process after it has performed the given trace, it may refuse). A process *divergence* is a *trace* such that, after performing that trace, it may livelock (i.e., it may perform an infinite sequence of internal actions, refusing all external events). The *traces* of a process is the set of all its traces. The *failures* of a process is the set of all its failures. The *divergences* of a process is the set of all its divergences. *Traces-refinement* is defined as this:

$$\texttt{SPEC} \sqsubseteq_T \texttt{IMPL} \mathrel{\hat{=}} traces(\texttt{IMPL}) \subseteq traces(\texttt{SPEC})$$

which reads $\texttt{SPEC} \sqsubseteq_T \texttt{IMPL}$: "a specification process `SPEC` is *traces-refined* by an implementation process `IMPL`".

Failures-refinement and *failures-divergence-refinement* are defined similarly:

$$\texttt{SPEC} \sqsubseteq_F \texttt{IMPL} \mathrel{\hat{=}} failures(\texttt{IMPL}) \subseteq failures(\texttt{SPEC})$$

$$\texttt{SPEC} \sqsubseteq_{FD} \texttt{IMPL} \mathrel{\hat{=}} (failures(\texttt{IMPL}) \subseteq failures(\texttt{SPEC})) \wedge$$

$$(divergences(\texttt{IMPL}) \subseteq divergences(\texttt{SPEC}))^3$$

A more in-depth discussion of traces, refinement and failures can be found in [Hoare 1985; Roscoe 1997; Schneider 1999].

[3] Note the direction of the $\sqsubseteq$, which might be a little counterintuitive.

Now consider the following ordering rule:

> After three elves have reported their arrival in the waiting room, Santa greets them and only them. The order in which Santa greets these elves may differ from the order in which they arrived. For each consultation, the arrival reports must happen before the greetings.

Here is a formalisation of this rule:

```
WG3 =
  |~| x:E_set @ report ! elfWaiting.x -> (
    |~| y:diff(E_set, {x}) @ report ! elfWaiting.y -> (
      |~| z:diff(E_set, {x,y}) @ report ! elfWaiting.z -> (
        |~| a:{x,y,z} @ report ! greet.a -> (
          |~| b:diff({x,y,z},{a}) @ report ! greet.b -> (
            |~| c:diff({x,y,z},{a,b}) @ report ! greet.c -> WG3
  ) ) ) ) )
```

`WG3` is a *sequential* process generating only `elfWaiting` reports and Santa `greet` reports. First, an elf with id `x` chosen from the set $\{0, \ldots, 9\}$ says he is waiting; this is followed by a waiting message from an elf with an id `y` chosen from the set $\{0, \ldots, 9\} \setminus \{\mathtt{x}\}$, followed by a waiting message from an elf with an id `z` chosen from the set $\{0, \ldots, 9\} \setminus \{\mathtt{x}, \mathtt{y}\}$. Now Santa greets an elf with id `a` from the set $\{\mathtt{x}, \mathtt{y}, \mathtt{z}\}$, then another greeting of an elf with id `b` from the set $\{\mathtt{x}, \mathtt{y}, \mathtt{z}\} \setminus \{\mathtt{a}\}$, and finally greets an elf with id `c` from the set $\{\mathtt{x}, \mathtt{y}, \mathtt{z}\} \setminus \{\mathtt{a}, \mathtt{b}\}$.

The `|~| e:S @ P` construction is a replicated internal choice. That is, the process `P` is executed with a value of `e` chosen arbitrarily from the set of elements of `S`. `diff(S, T) = S\ T` where `S` and `T` are sets.

To compare `SYSTEM_WR` against `WG3`, we must at least ensure trace equivalence. To do this, all reports other than `elfWaiting` and `greet` must be hidden (or commented out). Assume this has been done. FDR then confirms the following traces refinements: `SYSTEM_WR` $\sqsubseteq_T$ `WG3` and `WG3` $\sqsubseteq_T$ `SYSTEM_WR`. This means they are *traces equivalent.* Thus we can conclude that the two processes have exactly the same set of traces. In particular, there is no trace of `SYSTEM_WR` that violates the ordering rule.

Traces refinement does not allow us to conclude anything about the liveness of the systems analysed. For this, we need the failures model. FDR quickly verifies that `WG3` $\sqsubseteq_F$ `SYSTEM_WR`. This means that *any failure* of `SYSTEM_WR` is also a failure of the specification `WG3` (i.e., it is allowed). Turning this around: suppose `WG3` generates a trace, t, and is offered a set of events, S, that is not a refusal set (i.e., it will definitely accept one of them). Then, when `SYSTEM_WR` generates that same trace and is offered the same set of events, `SYSTEM_WR` will definitely proceed with one of them. It is *as alive* as the specification `WG3` – and never stalls.

We cannot compare `WG3` and `SYSTEM_WR` under the failures-divergences model because the hiding of all reindeer reports allows the latter to diverge – and `WG3` has no divergences. To make such comparison, `WG3` must be enhanced to allow the interleaving of at least one reindeer report (e.g. Santa's `allReindeer`) – and this must not be hidden in `SYSTEM_WR` (which eliminates its divergence).

We now have CSP formalisms for our Santa Claus system, verified for livelock and deadlock freedom and that a range of event ordering rules are obeyed. This now needs to be turned into executable code – a process that is, and has to be, trivial – see on-line Appendix E.

7. RELATED WORK

7.1 Other Implementations

As earlier described, a number of other implementations of possible solutions to the Santa Claus Problem exist.

The original solution by Trono himself [Trono 1994] was in Java; the corrected solution by Ben-Ari [Ben-Ari 1998] was in Ada.

The people behind Polyphonic C# [Benton et al. 2004] (now known as Cω) have demonstrated a solution [Benton 2003] using chords in Cω. Chords allow a method to have multiple headers, and only when all headers have been called will the body execute; this makes 'message passing' (and synchronisation) between multiple processes possible.

Simon Peyton Jones also uses the Santa Claus Problem as a running example in his chapter "Beautiful Concurrency" in the Beautiful Code book [Jones 2007]. Jones' solution is in Concurrent Haskell with *Software Transactional Memory.*

Other approaches using Actors with Multi-headed Message Receive Patterns [Sulzmann et al. 2008] (Actors, which preceded the notion of active objects, were originally proposed by Hewitt [Hewitt 1977]), Active C# [Güntensperger and Gutknecht 2004], and state classes [Cameron et al. 2006].

We have additional solutions ([Hurt and Pedersen 2008]) written in Java, C#, Groovy, C with Pthreads and C with MPI [Dongarra 1994]. These are available at www.santaclausproblem.net, together with a number of other occam-π implementations (including a symmetric version that does not use barriers, and a version using *process mobility*).

Note that Trono's description of the problem describes only the interactions between entities. It does not describe any external signalling, reflecting internal state changes, that are necessary to verify behaviour. Simon Peyton Jones' solution to the Santa Claus problem has only four external signals (i.e., print statements reporting reindeer delivering toys, elves consulting with Santa, Santa being woken by all the reindeer, and Santa being woken by a group of three elves). In our version of the system, we defined 17 different kinds of report, reflecting key state changes within the entities – and, then, specified a range of properties on those reports to be verified. See on-line Appendix F for an abbreviated occam-π solution using only the four signals described by Peyton Jones.

7.2 Model Checking and Formal Verification

Analysing (or developing) code by modelling aspects of interest in a formal specification language has been an area of immense interest over the years. Trusted tools that can automatically check for a range of standard and user-specifiable definitions of good behaviour are required. We mention a few of these here.

One often used formalism, when it comes to protocol verification, is Murϕ [Melton et al. 1996] (Examples include [Dill et al. 1992] and [Mitchell et al. 1997]). Murϕ can be used to describe a system of iterated guarded commands, much like the Unity language of [Chandy and Misra 1988]. A Murϕ specification is compiled to

C++ and linked with code for a verifier which checks for invariant violations, error statements, assertion violations, and deadlock [Dill et al. 1992]. This tool is based on Bryant's Ordered Binary Decision Diagrams (OBDD) [Bryant 1986], and like other tools for (formal) verification the state space explosion becomes a problem, but certain techniques do exist to help alleviate this [Ip and Dill 1996].

Another popular tool, also based on BBDs is νSMV [Cimatti et al. 2002], which allows specifications to be expressed in Computational Tree Logic (CTL) and Linear Temporal Logic (LTL), using BDD-based and SAT-based [Biere et al. 1999] model checking techniques.

A similar approach can be found in the SPIN [Holzmann 1997] model checker, which is based on Linear Temporal Logic (LTL), and uses the Promela (Process Meta Language) to specify the system to be verified. A new version of SPIN for checking nonblocking MPI programs (mpiSpin) is also being developed [Siegel 2007].

A different approach to formal verification (or at least deadlock detection) is Petri nets and colouring games [Huber et al. 1985; Jensen 1997] where systems are modelled by graphs whose nodes can contain tokens, which are transferred to other nodes along the arcs of the graph according to certain logic rules. In order to determine deadlock-freedom in coloured Petri nets, an NP-complete task, an occurrence graph is constructed – an occurrence graph represents all possible markings, that is, all possible configurations of the Petri net, and strongly connected components are computed. If a strongly connected component exists that does not have any arcs leaving it, then a configuration exists that represents either a livelock or a deadlock. Techniques for reducing the size of the occurrence graph exist; even so, the size of the graphs and the time to evaluate them can both be exponential.

8. REFLECTIONS

8.1 Process Orientation

This paper presents a *process-oriented* solution to the Santa Claus problem. Initially, the overall design is laid out through process network diagrams, whose nodes represent processes and edges show allowed synchronisations (data flow and barriers). Executable occam-π code and model-checkable CSP script are then developed, with each process being produced independently (thanks to the compositionality of the underlying semantics) and the network descriptions derived directly from the diagrams. State information in this system is sufficiently small so that no state reducing abstractions are needed in the CSP version[4]. This means that the occam-π and CSP representations are in 1-1 correspondence and that development and maintenance can be performed in whichever form is most convenient.

8.2 Formal Verification

We claim that process-oriented design leads to solutions that directly reflect natural structures in the problem space and that, therefore, our confidence in these solutions is high – they have an element of *obvious correctness* that makes us feel comfortable. However, we have both executable and model checkable forms so that this confidence can be formally explored and reinforced. Occasionally, when verification shows

[4]A single unconstrained 32-bit integer potentially introduces four gigastates!

our confidence to have been misplaced (e.g. Section 6.1), corrections are simple to perform and (re-)verify (e.g. Sections 6.2 and 6.3).

At least for Santa Claus, the evidence is this paper supports the above claim. The occam-π code worked first time – once it had passed the stringent safety rules checked by the compiler. Correct output appears and the system has never been seen to deadlock. Applying the FDR model checker to the CSP script indeed verifies the absence of deadlock and other bad behaviours, such as livelock. The solution is also free from data race hazards (unsynchronised access to shared information): such hazards do not exist in CSP or occam-π – they have no expression.

Other constraints, specific to the additional reporting we have added to the requirements, have been formalised and verified. Such signals provide external evidence of internal activity. They can be used to drive an animation of that activity or, perhaps, to control the operation of some complex machinery (whose rules of operation are modelled by the system). If such machines are safety critical and inappropriate (e.g. wrongly ordered) control signals would lead to breakdown, verification is essential.

8.3 The Santa Claus Experience

An example of the formal specification and verification of signal ordering rules is shown in Section 5.5. The rule is directly modelled by a CSP process, `CHECK_A`, observing the system reports. This observer provokes deadlock (by simply stopping and, hence, refusing further reports) if it sees a rule violation. However, the model checker verifies deadlock-freedom of the parallel observer-observed system. Therefore, we may deduce that the observed system always honours the rule.

A maintenance change, introducing further reports showing detail of more internal states, was described in Section 6.1. Perhaps these signals were needed to drive a more sophisticated animation (or machine). Here was an example where our confidence in the "obvious correctness" of our design was dented. The original reporting rule (whose verification still, of course, holds in the modified system) was itself modified by replacing some of the old reports with the new ones.

Although we expected the rule to be honoured – and we never saw it dishonoured in any run of the executable – the model checker immediately reported the potential for violation and told us exactly how it could occur! Armed with this information, the CSP script was easily changed to eliminate that particular problem. Formal verification then confirmed that the corrected system now obeyed the new rule, along with all the old rules. Because of the direct structural correspondence between the CSP script and occam-π code, this correction was trivially carried over to the executable form (on-line Appendix E). A similar development was described in Section 6.3, although this time we had anticipated the need for system correction.

A different approach to rule verification is described in Section 6.4. This time, a (highly) non-deterministic process is built to generate only the signals mentioned in the ordering rule and, explicitly, to honour that rule – this becomes the formal specification of the rule. The model checker then verifies that our system, with all signals other than those in the rule hidden, *failure-refines* that specification. This means that our system is as *alive* as the specification – this is, after any trace of its execution, the implementation is able to perform anything that the specification demands.

8.4 Some Other Considerations

There is one aspect of our solution that we have not verified: Santa's prioritisation of the reindeer wake-up signal over that from a group of elves. Our implementation is direct and trivial, using the `PRI ALT` construct of occam-π (Sections 3.4 and 4.1). Our CSP formalisation maps this to a plain *external choice* (Section 5.3). CSP does not address priorities, so we have no direct way to express this part of the specification formally. We can, however, express this indirectly (and verify) using an additional *priority managing process* with which Santa, the reindeer and the elves must all engage. This is discussed in the on-line Appendix G.

We note that, even though switching between executable occam-π code and model-checkable CSP script is not difficult, it would be simpler to deal with only one formalism. Prospects for this are discussed in on-line Appendix H.

Finally, the solutions developed in this paper work with only the static network mechanisms of occam-π. All connections between processes (whether through channels or barriers) are laid down in advance. They are always available to the processes and care must be taken that they are used only at appropriate times.

Mobile processes, along with mobile channels and barriers, enable process networks to be dynamic: they may change their size (number of processes, channels, barriers) and shape (connection topology) as they run – much like living organisms. One of the benefits is that all connections do not have to be established statically, in advance of when they are needed and open to abuse. This is discussed in on-line Appendix I.

ELECTRONIC APPENDIX

The electronic appendix for this article can be accessed in the ACM Digital Library by visiting the following URL: `http://www.acm.org/pubs/citations/journals/toplas/2010-32-4/p1-welch`.

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Santa Claus: Formal Analysis of a Process-Oriented Solution

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A. STATES AND REPORTS

The Santa Claus system has three kinds of component: Santa, reindeer and elf. Their states are outlined below, along with the reports we require them to make as they cycle through them. Most of these outline 'states' are a composite of a short sequence of states. Some allow many alternative sequences (e.g. when harnessing the reindeer, the order in which they are harnessed does not matter). The reports are triggered by internal state changes – they signal their occurrence to the system environment (which may be a machine controlled by these signals). The state changes are triggered by event synchronisations between the system processes, hidden from their environment. Those events are not specified here – their design, together with any additional components deemed necessary, is an implementation matter (Section 4).

- **Santa**
  - **States:**
    - *Sleeping* — wait to be woken up either by all the reindeer ready for harnessing or a group of three elves wanting an audience;
    - *Delivering* — harness all nine reindeer, deliver toys, unharness the reindeer, back to sleep;
    - *Consulting* — greet the elves party, consult with them, show them out, back to sleep.
  - **Reports:**
    - who woke him up (reindeer or elves);
    - harnessing each reindeer;
    - start of delivery round;
    - end of delivery round;
    - unharnessing each reindeer;
    - greeting each elf;
    - start of consultancy session;

    —end of consultancy session;
    —showing each elf the door.

—**Reindeer (nine of them)**
  —**States:**
    —*Vacationing* – do nothing until bored, then go home;
    —*Waiting* – wait for the other reindeer to come home, then approach Santa;
    —*Delivering* – get harnessed, wait for the other reindeer to be harnessed, deliver toys, get unharnessed, then back on holiday;
  —**Reports:**
    —start of vacation postcard;
    —returning home email;
    —start of toy delivery message;
    —return from toy delivery.

—**Elf (ten of them)**
  —**States:**
    —*Working* – work until a problem (or new idea) arises, then go to see Santa;
    —*Waiting* – wait for two other elves who also want to see Santa, approach Santa as a group of three (this must be the *only* such group doing so);
    —*Consulting* – greet Santa, wait for the other elves to greet Santa, consult together, say goodbye to Santa, then back to work;
  —**Reports:**
    —starting work shift;
    —when a work problem (or idea) occurs;
    —consulting with Santa;
    —consultancy session finished.

B. MESSAGE PROTOCOLS – OCCAM-π

The following declarations give the message structures for the reindeer, elf and Santa reports used by the occam-π processes in Section 4.

```
PROTOCOL REINDEER.MSG
  CASE
    holiday; INT         -- start of vacation postcard; reindeer id
    deer.ready; INT      -- back from vacation; reindeer id
    deliver; INT         -- start of toy delivery; reindeer id
    deer.done; INT       -- return from toy delivery; reindeer id
:

PROTOCOL ELF.MSG
  CASE
    working; INT         -- start of work shift; elf id
    elf.ready; INT       -- want to consult Santa; elf id
    consult; INT         -- consulting; elf id
    elf.done; INT        -- end of consultation; elf id
:
```

```
PROTOCOL SANTA.MSG
  CASE
    reindeer.ready       -- woken up by reindeer
    harness; INT         -- harnessing this reindeer; id
    mush.mush            -- start of toy delivery
    woah                 -- end of toy delivery
    unharness; INT       -- unharnessing this reindeer; id
    elves.ready          -- woken up by party of elves
    greet; INT           -- greet this elf; id
    consulting           -- consulting with elves
    santa.done           -- end of consultation
    goodbye; INT         -- show elf the door; id
:

PROTOCOL MESSAGE EXTENDS REINDEER.MSG, ELF.MSG, SANTA.MSG:
```

C. EXTENDED PARTIAL BARRIER – OCCAM-π

The following code declares the occam-π process for an extended partial barrier, as described in Section 4.2.2.

```
PROC xp.bar (VAL INT x, CHAN BOOL a?, b?, ping!)
  WHILE TRUE
    SEQ
      SEQ i = 0 FOR x    -- gather in the right number of offers
        BOOL any:
        a ? any
      ping ! TRUE        -- trigger some special process
                         --   (enough clients are ready)
      SEQ i = 0 FOR x    -- complete partial barrier
        BOOL any:        --   (let the clients go)
        b ? any
:
```

D. CSP SCRIPTS

D.1 Santa Claus Events

First, we declare all the constants, types and events used by the system:

```
N_REINDEER = 9      -- number of reindeer
G_REINDEER = 9      -- number of reindeer needed to meet Santa

N_ELVES = 10        -- number of elves
G_ELVES = 3         -- number of elves needed to meet Santa

nametype R_set = {0..(N_REINDEER - 1)}         -- reindeer ids
nametype E_set = {0..(N_ELVES - 1)}            -- elf ids

datatype ReportTag =
  holiday | deerReady | deliver | deerDone |   -- reindeer reports
  working | elfReady | elfWaiting |            -- elf reports
  consult | elfDone |                          --   (more of above)
```

```
    allReindeer | harness | mushMush |           -- santa’s reindeer reports
    woah | unharness |                           --   (more of above)
    threeElves | greet | consulting |            -- santa’s elf reports
    done | goodbye                               --   (more of above)

  channel report : ReportTag.E_set               -- report.id
                                                 --   (E_set includes R_set)

  channel reindeer_2_santa : R_set               -- reindeer id
  channel reindeer_2_santa_ack

  channel elves_2_santa : E_set                  -- elf id
  channel elves_2_santa_ack

  channel just_elves_a, just_elves_b             -- extended partial barrier
  channel just_elves_ping                        --   (extension channel)
  channel santa_elves_a, santa_elves_b           -- partial barrier

  channel just_reindeer, santa_reindeer          -- barriers
```

D.2 Partial Barriers

Like occam-π, CSP has no primitives for partial barriers and we have to model them using processes and events. Here is the regular partial barrier process, again binding in the specific events for this system. It directly follows the occam-π code given in Section 4.2.1:

```
  P_BAR (n) =
    (; x:<1..n> @ (santa_elves_a -> SKIP)) ;
    (; x:<1..n> @ (santa_elves_b -> SKIP)) ; P_BAR (n)
```

Note: the syntax `(; x:<1..n> @ P(x))` indicates a *replicated sequence*. It means: `P(1); P(2); ... ; P(n)`.

The following CSP expression corresponds to the `xp.bar` occam-π process described in Section 4.2.2 (and shown in Appendix C):

```
  XP_BAR (n) =
    (; x:<1..n> @ (just_elves_a -> SKIP)) ;
    just_elves_ping ->
    (; x:<1..n> @ (just_elves_b -> SKIP)) ; XP_BAR (n)
```

D.3 Reindeer

We define the reindeer process directly from the occam-π declaration in Section 4.1:

```
  REINDEER (id) =
    report ! holiday.id -> RANDOM_TIMEOUT (HOLIDAY_TIME) ;
    report ! deerReady.id -> just_reindeer ->
    reindeer_2_santa ! id -> santa_reindeer ->
    report ! deliver.id -> santa_reindeer ->
    report ! deerDone.id -> reindeer_2_santa ! id ->
    reindeer_2_santa_ack -> REINDEER (id)
```

D.4 The Santa Claus System

We build this up a few processes at a time. From Figure 2, we see that the reindeer *interleave* on the `report` and `reindeer_2_santa` channels, but must *synchronise* on the `just_reindeer` and `santa_reindeer` barriers. To express this, simply omit `report` and `reindeer_2_santa` from the synchronisation set of the parallel operator[5]:

```
ALL_REINDEER =
  ([| {just_reindeer, santa_reindeer} |] id:R_set @ REINDEER (id))
  \ {just_reindeer}
```

Note: the `just_reindeer` event is hidden in the above, since it is a matter of concern *only* to the reindeer.

With the partial barriers represented by the shared channels and processes of Figure 6, the elf processes do not synchronise directly with each other at all – they interleave on all their channels:

```
ALL_ELVES = ||| id:E_set @ ELF (id)
```

In CSP (and occam-π), concurrency is associative and commutative – so, it does not matter in which order we construct the full system. Following Figure 6, we combine the elves with their extended partial barrier process:

```
XP_ELVES =
  ( ALL_ELVES [| {just_elves_a, just_elves_b} |] XP_BAR (G_ELVES) )
  \ {just_elves_a, just_elves_b}
```

and add in Santa:

```
SANTA_XP_ELVES =
  ( SANTA
      [| {| just_elves_ping, elves_2_santa, elves_2_santa_ack |} |]
    XP_ELVES
  ) \ {| just_elves_ping, elves_2_santa, elves_2_santa_ack |}
```

and add in the (non-extended) partial barrier:

```
SANTA_XP_P_ELVES =
  ( SANTA_XP_ELVES
      [| {santa_elves_a, santa_elves_b} |]
    P_BAR (G_ELVES + 1)
  ) \ {santa_elves_a, santa_elves_b}
```

Finally, we have the whole system:

```
SYSTEM =
  ( SANTA_XP_P_ELVES
      [| {| santa_reindeer, reindeer_2_santa, reindeer_2_santa_ack |} |]
    ALL_REINDEER
  ) \ {| santa_reindeer, reindeer_2_santa, reindeer_2_santa_ack |}
```

Note that all events are now hidden *except* for the `report` channel.

[5]The CSP parallel operator used here consists of a *synchronisation set* of events embedded between the symbols [| and |]. Operand processes cannot engage in events from that set independently – all must engage for the event to happen. If the curly brackets of the synchronisation set also include bars (i.e., {| and |}), the set contains *all* the messages carried by the channels named.

E. BACK TO OCCAM-π

Sections 6.1, 6.2 and 6.3 modify the CSP formalisms for our Santa Claus system derived from the occam-π executable code presented is Section 4. These modifications address additional event ordering rules on additional reports providing more detailed information on the behaviour of the system. The modifications were verified for correctness and deadlock and livelock freedom. They need to be turned back into executable code – a process that is, and has to be, trivial:

—a `waiting.room` process replaces the extended partial barrier `xp.bar` – this corresponds to the CSP process `WR` (Section 6.3);
—this `waiting.room` needs an extra acknowledge channel for `just_elves_a_ack` and uses it as specified by `WR`;
—the `elf` process also needs to be modified to synchronise on this acknowledgement – as specified by the CSP process `ELF` (Section 6.2);
—the `waiting.room` also needs to perform the second synchronisation needed on its `ping` channel at the end of its cycle;
—the `santa` process accepts this second `ping` after making its welcome report and greeting the elf consulting group – as specified by `SANTA` (Section 6.3).

Here is the waiting room (modified from `xp.bar` is Section 4.2.2, following `WR` from Section 6.3):

```
  PROC waiting.room (VAL INT x, CHAN BOOL a?, a.ack?, b?, ping!)
    WHILE TRUE
      SEQ
        SEQ i = 0 FOR x      -- gather in the right number of elves
          BOOL any:
          SEQ
            a ? any          -- let an elf in
            a.ack ? any      -- wait for the elf to report its entry
        ping ! TRUE          -- try to wake up Santa
                             --   (enough elves are here)
        SEQ i = 0 FOR x      -- let the elves leave the waiting room
          BOOL any:
          b ? any
        ping ! TRUE          -- wait for Santa to greet all the elves
  :
```

The elf process needs the extra acknowledgement channel for its interaction with the waiting room. This interaction is more than the partial barrier protocol previously employed (the double output implemented by the `sync` process). These outputs must now sandwich the report (that the elf is in the waiting room) and the acknowledgement. The changes are coded in-line below (modified from `elf` in Section 4.2.3 and following `ELF` from Section 6.2):

```
PROC elf (VAL INT id,
          SHARED CHAN BOOL just.elves.a!, just.elves.a.ack!, just.elves.b!,
          SHARED CHAN BOOL santa.elves.a!, santa.elves.b!,
          SHARED CHAN INT to.santa!,
          SHARED CHAN ELF.MSG report!)
```

```
  WHILE TRUE
    SEQ
      ...
      CLAIM report ! elf.ready; id    -- "I want to see Santa" + id
      CLAIM just.elves.a ! TRUE       -- let me into the waiting room
      CLAIM report ! waiting; id      -- "I'm in the waiting room" + id
      CLAIM just.elves.a.ack ! TRUE   -- you can let someone else in now
      CLAIM just.elves.b ! TRUE       -- wait for enough elves to gather
      CLAIM to.santa ! id             -- say hello to Santa
      ...
:
```

Finally, Santa needs to accept the second `ping` from the waiting room. We modify `santa` from Section 4.2.3, following `SANTA` from Section 6.3:

```
  {{{  deal with the elves
  BOOL any:
  just.elves.ping ? any               -- a party of elves is at the door
    SEQ
      CLAIM report ! elves.ready      -- "Ho, Ho, Ho, some elves are here"
      SEQ i = 0 FOR G.ELVES           -- for each elf in the consult party
        INT id:                       -- (G.ELVES is size of party)
        SEQ
          from.elves ? id             -- receive elf id
          CLAIM report ! greet; id    -- "Hello elf " + id
      just.elves.ping ? any           -- tell waiting room all have arrived
      CLAIM report ! consulting       -- "Consulting with elves"
      ...
  }}}
```

F. A SIMPLER SANTA CLAUS

For completeness and comparison reasons, we have included the occam-π version of the program that generates the same output as the Peyton Jones' implementation (in Haskell using Software Transactions [Jones 2007]). As described in Section 7.1, the latter solution has only four external signals: reindeer delivering toys, elves consulting with Santa, Santa being woken by all the reindeer, and Santa being woken by a group of three elves.

```
PROC reindeer (VAL INT id, BARRIER just.reindeer, santa.reindeer,
               SHARED CHAN INT to.santa!,
               SHARED CHAN REINDEER.MSG report!)
  WHILE TRUE
    SEQ
      random.wait (HOLIDAY.TIME)  -- sleep for a random amount of time
      SYNC just.reindeer          -- wait for all deer to return
      CLAIM to.santa ! id         -- send id to Santa (to get harnessed)
      CLAIM report ! deliver; id  -- "I'm delivering toys" + id
      SYNC santa.reindeer         -- until Santa takes us all home
:
```

```
PROC elf (VAL INT id,
          SHARED CHAN BOOL just.elves.a!, just.elves.b!,
          SHARED CHAN BOOL santa.elves.a!, santa.elves.b!,
          SHARED CHAN INT to.santa!, SHARED CHAN ELF.MSG report!)
  WHILE TRUE
    SEQ
      random.wait (WORKING.TIME)           -- work until I have a problem
      sync (just.elves.a, just.elves.b)    -- wait for two other elves
      CLAIM to.santa ! id                  -- say hello to Santa
      CLAIM report ! consult; id           -- "I'm consulting Santa" + id
      sync (santa.elves.a, santa.elves.b)  -- until Santa has had enough
:

PROC santa (CHAN INT from.reindeer?, BARRIER santa.reindeer,
            CHAN BOOL just.elves.ping?, CHAN INT from.elves?,
            SHARED CHAN BOOL santa.elves.a!, santa.elves.b!,
            SHARED CHAN SANTA.MSG report!)
  WHILE TRUE
    PRI ALT
      INT id:
      from.reindeer ?? id                  -- the first reindeer is here
        CLAIM report ! reindeer.ready      -- "Ho, Ho, Ho, reindeer are here"
        SEQ                                -- (extended input has finished)
          SEQ i = 0 FOR N.REINDEER - 1     -- for each remaining deer
            from.reindeer ? id             --   receive deer id
          random.wait (DELIVERY.TIME)      -- deliver toys for a random time
          SYNC santa.reindeer              -- signal everyone to return home
      BOOL any:
      just.elves.ping ? any                -- a party of elves is at door
        SEQ
          CLAIM report ! elves.ready       -- "Ho, Ho, Ho, elves are here"
          SEQ i = 0 FOR G.ELVES            -- for each elf in the party
            INT id:
            from.elves ? id                --   receive elf id
          random.wait (CONSULTATION.TIME)  -- consult for a random time
          sync (santa.elves.a, santa.elves.b)   -- consultancy is over
:
```

Only processes *different* from those presented previously are listed above. The processes managing the partial barriers, `p.bar`, `xp.bar` and `sync` are as defined in Sections 4.2.1 and on-line Appendix C. The whole system, `santa.system`, remains as defined in Section 4.3.

G. PRIORITISED CHOICE

There is one aspect of our solution that we have not verified: Santa's prioritisation of the reindeer wake-up signal over that from a group of elves. The implementation is direct and trivial, using the `PRI ALT` construct of occam-π (Sections 3.4 and 4.1). Our CSP formalisation maps this to a plain *external choice* (Section 5.3). CSP does not address priorities, so we have no direct way to express this part of the specification formally.

We can, however, express it indirectly, using an event-ordering observer as before. This observer takes note only of the reindeer reports saying they are back from holiday and Santa's "Ho, Ho, Ho ..." messages greeting either the reindeer or the elves party. The observer keeps a count of reindeer reports, clearing this to zero upon the relevant Santa greeting. If Santa greets an elves party when the reindeer count is nine, the observer refuses all further signals, provoking system deadlock.

However, were we to do this, the model check would show the potential for deadlock – that is, our system does not honour the rule:

> Santa never says "Ho Ho Ho ... some elves are here" **if** all nine reindeer have reported they are back from holiday **and** Santa has not since said "Ho Ho Ho ... Some reindeer are here".

This rule, however, is different from the rule actually required, which is that Santa gives priority to the reindeer wake-up signal (and *is* honoured).

There is a difference between the back-from-holiday reports from the reindeer and the wake-up signal to Santa: the former *precede* the latter! When the ninth reindeer reports the end of her holiday, the signal to Santa will certainly be made ... but the offer of that signal *follows* that last report and is not *atomic* with it. In fact, there is even a barrier synchronisation of all the reindeer in between them – though that is not relevant. What is relevant is that a group of three elves may signal Santa following the ninth back-from-holiday reindeer report and that Santa can quite properly accept that elves signal (since the reindeer signal has not yet been offered). These circumstances are rare, but we have observed them when running the occam-π code.

Whenever Santa actually has a choice of wake-up signals, the occam-π system will ensure Santa makes the right choice. These circumstances are quite common: we only need a long consultancy with one party of elves, during which another party assembles and all the reindeer return. Our solution honours the informal requirement regarding prioritised choice but we have no way (in CSP) to formalise and, hence, verify it.

If we want to honour the above event-ordering rule instead, of course we can. We need a *priority managing process* (PMP) to which Santa enquires for a decision as to which wake-up signal to process (and sleeps until given an answer). The reindeer and party of elves must register with PMP *after* their last member has arrived but *before* reporting that arrival.

On the elves side, this is easily managed by the *Waiting Room* process. For the reindeer, this is not so easy since the last reindeer back from holiday does not know that she is the last! To manage this, simplest is to abandon the `just.reindeer` barrier synchronisation and replace it with a (full) barrier process – similar to the waiting room (perhaps it is their *Stable*).

The PMP must always service such registration without blocking. It can be a simple server offering an external choice between an enquiry from Santa or registration from the reindeer or elves. No prioritisation or fairness is required for this service.

If a Santa enquiry is outstanding when a registration comes in, Santa is simply told to take the signal from the registering side (that will soon be made, following a report that it is about to make it). Otherwise, the registration is just noted (and the registering side goes on to report that it is ready to signal and, then, offers its signal to Santa – which, for now, will be blocked).

If no registrations have been made when Santa enquires, the enquiry is noted (leaving Santa blocked waiting for a reply). If one registration exists when Santa enquires, Santa is informed of the registering side and the registration is cleared. If both registrations exist, Santa is told to accept the reindeer signal and that registration is cleared.

Finally, Santa accepts the reply from the PMP with an *extended* rendezvous, during which the wake-up signal (from the instructed side) is accepted and the "Ho, Ho, Ho" greeting is reported. This prevents the PMP from accepting a further registration (which would trigger a report of an impending signal) before Santa's report of its decision has been made.

The PMP logic is easier to express in occam-π or CSP. The modifications to the rest of the system (to replace the `just.reindeer` barrier with a *Stable* process and simple changes to the *Waiting Room* logic to register with the PMP when the third elf arrives and before it reports that arrival), the PMP itself, the observer process checking the rule given in this section and the model checking to verify the rule are omitted for reasons of space. However, all are straightforward and available from the website www.santaclausproblem.net (along with all occam-π codes and CSP scripts in this paper).

H. A UNIFIED OCCAM-π MODEL CHECKER

Building CSP models automatically from occam-π is tractable. (Even the mobile channels, barriers and processes of occam-π, though not used here, have an operational semantics in terms of standard CSP [Welch and Barnes 2005b; 2008].) For producing scripts that do not generate unnecessarily large state spaces when model checking, a transformation tool allowing programmer interaction (to roll states together, carefully introduce non-determinism) would be helpful – although that was not needed for this Santa Claus system.

Building *efficient* executables automatically from CSP implementation scripts (i.e., those with internal choices resolved) is a little harder – especially if the full range of CSP expression (e.g. external choice over multiway synchronisations [Welch et al. 2006; Welch 2006], complex choice preconditions on channel message values) is allowed.

Either way, being able to translate automatically between these two formalisms unifies verification with the development of efficiently executable programs – one of the grand challenges in Computer Science. We look forward to developments in both these areas.

I. MOBILE CHANNELS AND PROCESSES

In the solution analysed in the paper, network topology is static – all processes have all their channels available all the time. Some care has to be taken not to use channels when inappropriate (e.g. a reindeer may try to communicate with Santa while on vacation). With occam-π mobiles, we can arrange that processes simply do not have those channels until they get into the right state – and not having such channels means that their misuse cannot even be expressed! Of course, it is a natural consequence of mobile system design that the arrivals of channels (or barriers or processes) are the very events triggering their use.

In occam-π, we can construct channels at run-time and pass their ends *independently* over existing channels to create new topology. For the Santa Claus problem, moving a channel-end from an elf (or reindeer) through the waiting room (or stable) and on to Santa, whilst retaining the other end, establishes a connection (between elf and Santa, or reindeer and Santa) when, and only when, it is needed.

In occam-π, we can construct processes (to well-defined plugin templates) at run-time, send and receive them through channels, plug them into local networks, fire them up, stand them down and move them on again. We can model the reindeer and elves as mobile processes that move through holiday resorts, stables, work benches, waiting rooms, Santa's Grotto and back again. All those destinations are also processes though static ones. As the reindeer and elves arrive at each stage, they plug in and do business.

We sense that these solutions are even more realistic and easy to design, program and reason about, at least informally, than the static solutions expressed here. Indeed, some of the careful extra synchronisations, added to the static network solution presented here to prevent out-of-order external signalling, are not needed (e.g. an elf only reports that it is in the waiting room *when it is in the waiting room* and before it knocks on Santa's door). Thus, the solutions become simpler.

For formal reasoning, an operational semantics maps occam-π mobile channels and barriers down to CSP [Welch and Barnes 2005b; 2008]. We believe a similar approach will yield an operational semantics for mobile processes in occam-π. More recently, Roscoe has shown [Roscoe 2009] that CSP+ (which is CSP with the addition of an exception-throwing operator) can simulate a rich set of new capabilities, including the mobility features of the π-calculus. This promises a direct traces-failures-divergences semantics for occam-π, for which new model checkers may be built with direct ability for reasoning about mobility.

We will leave these approaches for another time. However, occam-π solutions employing them will be placed on www.santaclausproblem.net.